

Dipolar exchange induced transparency with Rydberg atoms

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A three-level atomic medium can be made transparent to a resonant probe field in the presence of a strong control field acting on an adjacent atomic transition to a long-lived state, which can be represented by a highly excited Rydberg state. The long-range interactions between the Rydberg state atoms then translate into strong, non-local, dispersive or absorptive interactions between the probe photons, which can be used to achieve deterministic quantum logic gates and single photon sources. Here we show that long-range dipole-dipole exchange interaction with one or more spins – two-level systems represented by atoms in suitable Rydberg states – can play the role of control field for the optically-dense medium of atoms. This induces transparency of the medium for a number of probe photons n_p not exceeding the number of spins n_s , while all the excess photons are resonantly absorbed upon propagation. In the most practical case of a single spin atom prepared in the Rydberg state, the medium is thus transparent only to a single input probe photon. For larger number of spins n_s , all $n_p \leq n_s$ photon components of the probe field would experience transparency but with an n_p -dependent group velocity.

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I. INTRODUCTION.

Atoms excited to the Rydberg states with high principal quantum numbers $n \gg 1$ have very long natural lifetimes $\tau \propto n^3$ and strong electric dipole moments $\varphi \propto n^2$ for the microwave transitions between the neighboring states [1]. The resulting long-range, resonant (exchange or Förster) and nonresonant (dispersive or van der Waals) dipole-dipole interactions between the atoms can suppress multiple Rydberg excitations within a certain blockade distance [2–5]. Dipole-dipole exchange interactions can mediate long-range binding potentials between Rydberg atoms [6–8] and can be used to study coherent [9–11] and incoherent [12, 13] excitation transfer processes.

An optically-dense atomic medium can be made transparent to a resonant probe field whose photonic excitations are coherently mapped onto the atomic excitations, forming the so-called dark-state polaritons [14, 15]. This effect is called electromagnetically induced transparency (EIT) [15], and it is usually mediated by a control laser field driving the atoms on the transition adjacent to the probe resonance. Alternatively, the driving laser can be replaced by an electromagnetic mode of a resonator strongly coupled to the corresponding atomic transition [16, 17]. For an initially empty cavity, the resulting vacuum induced transparency (VIT) is sensitive to the number of photons in the input probe pulse and can therefore serve as a photon-number filter [18].

Here we propose a hitherto unexplored mechanism to attain transparency for a weak resonant probe field propagating in an ensemble of atoms whose adjacent transition is strongly coupled by dipole-dipole exchange interaction to one or more spins – two-level systems – playing the role of a quantized control field. In analogy with EIT and VIT, we call this mechanism dipolar exchange induced transparency – DEIT. By employing resonant

dipole-dipole interaction between suitable pairs of highly-excited Rydberg states, we ensure that the atoms of the medium are subject to a strong and long-range dipolar exchange field of the effective spins, see Fig. 1. Each probe photon propagating in the DEIT medium with a slow group velocity creates an accompanying Rydberg excitation by flipping one spin. The number of spins n_s then determines the maximal number of probe photons $n_p \leq n_s$ that can simultaneously be accommodated in the medium without absorption. Once all the spins are flipped, the excess $(n_p - n_s)$ photons see resonant two-level atomic (TLA) medium. If the medium is optically thick, it absorbs all of the excess photons. The system can thus serve as a photon-number filter, with the number of appropriately prepared spins $n_s = 0, 1, \dots$ being the switch.

We note related but conceptually different studies of Rydberg EIT with atoms in a ladder configuration of levels [19–40]. These schemes employ essentially conventional EIT for the probe field acting on the atomic transition between the ground $|g\rangle$ and intermediate excited $|e\rangle$ states with a classical driving field coupling state $|e\rangle$ to a high-lying Rydberg state $|r\rangle$. In such a medium, the probe photons turn into dark-state polaritons having large admixture of atomic Rydberg excitations. The interactions between the Rydberg-state atoms then lead to strong, non-local interactions between the photons. In particular, Rydberg mediated interactions can result in large conditional phase shifts [19–24], or even more dramatically, destruction of transparency of the medium within a blockade distance of d_b around a single propagating or stored Rydberg polariton [24, 25, 35–38]. But complete scattering of light induced by a single Rydberg excitation requires large optical depth per blockade distance $d_b \lesssim 10 \mu\text{m}$, which entails problems: Increasing the atom density and/or choosing higher Rydberg states $n \gtrsim 100$ to increase the range of the van der Waals inter-

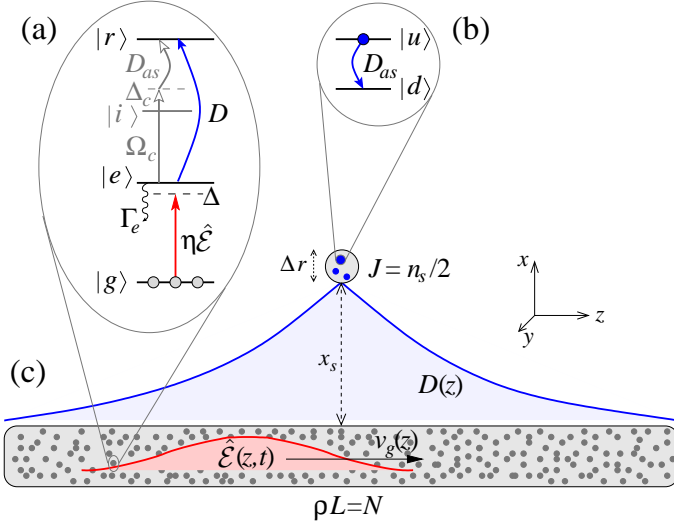


FIG. 1. Schematics of the system. (a) Level configuration of atoms interacting with the probe field $\hat{\mathcal{E}}$ on the transition between the ground state $|g\rangle$ and excited state $|e\rangle$ which decays with rate Γ_e , while the coupling D to the Rydberg state $|r\rangle$ is mediated by the dipole-dipole exchange interaction D_{as} with the effective spin- J (atoms in (b)) and an auxiliary laser Ω_c detuned from the non-resonant intermediate Rydberg state $|i\rangle$ by $\Delta_c \gg \Omega_c, D_{as}$. (b) The $n_s \geq 1$ atoms with the Rydberg states $|u\rangle$ and $|d\rangle$, confined in a small volume of size Δr , form an effective spin $J = n_s/2$ which interacts with the medium atoms in (a) via the dipole-dipole exchange. (c) The probe pulse $\hat{\mathcal{E}}(z, t)$ propagates with group velocity $v_g(z)$ along the z axis in an optically dense atomic medium of linear density ρ and length L . The atoms at positions z are subject to dipole-dipole interaction $D(z) \equiv D(z\mathbf{e}_z - \mathbf{r}_s)$ with the effective spin J at position \mathbf{r}_s resulting in DEIT for the probe field.

actions leads to strong decoherence of the Rydberg-state electrons [41] and thereby EIT inhibition.

In the present scheme, the situation is in a sense reversed: A single spin – atom in a suitable Rydberg state with large transition dipole moment – does induce transparency of the medium for a single probe photon within the interaction distance d_t . Due to the longer range of the resonant dipole-dipole interaction, as compared to the van der Waals interaction, DEIT with atoms excited to lower Rydberg states with $n \sim 80$ can extend over the medium of length $L = 2d_t \simeq 25 \mu\text{m}$. The excess photons are then completely scattered upon propagation in the medium with moderate atom density but large optical depth.

II. MATHEMATICAL FORMALISM

A. The Hamiltonian of the system

We now turn to the quantitative description of the system shown schematically in Fig. 1. Consider a one-

dimensional propagation and interaction of a weak (quantum) probe field $\hat{\mathcal{E}}(z, t)$ with the atomic medium of linear density $\rho(z)$ and length L , taken as quantization length. In the interaction picture [42, 43], the Hamiltonian of the system reads

$$\begin{aligned} H/\hbar = & -i\frac{c}{L} \int dz \hat{\mathcal{E}}^\dagger(z) \partial_z \hat{\mathcal{E}}(z) \\ & - \int dz \rho(z) [\Delta \hat{\sigma}_{ee}(z) + (\Delta + \delta) \hat{\sigma}_{rr}(z) \\ & \quad + (\eta \hat{\mathcal{E}}(z) \hat{\sigma}_{eg}(z) + \text{H.c.})] \\ & + \int dz \rho(z) [\hat{\sigma}_{re}(z) \sum_j^{n_s} D(z\mathbf{e}_z - \mathbf{r}_j) \hat{\sigma}_-^{(j)} + \text{H.c.}] \end{aligned} \quad (1)$$

Here the first term is the free Hamiltonian for the probe field $\hat{\mathcal{E}}(z) = \sum_k \hat{a}_k e^{ikz}$ propagating with velocity c . The probe field operators obey the commutation relations $[\hat{\mathcal{E}}(z), \hat{\mathcal{E}}(z')] = [\hat{\mathcal{E}}^\dagger(z), \hat{\mathcal{E}}^\dagger(z')] = 0$ and $[\hat{\mathcal{E}}(z), \hat{\mathcal{E}}^\dagger(z')] = L\delta(z - z')$ which follow from the bosonic nature of operators $\hat{a}_k, \hat{a}_k^\dagger$ for the individual longitudinal modes k . The second term of Eq. (1) describes the atoms of the medium and their interaction with the probe field $\hat{\mathcal{E}}$ with the coupling strength $\eta = \wp_{ge} \sqrt{\omega/(2\hbar\epsilon_0 w^2 L)}$, where \wp_{ge} is the dipole matrix element of the transition $|g\rangle \rightarrow |e\rangle$, ω is the carrier frequency of the probe field, ϵ_0 is the vacuum permittivity, and $w \ll L$ is the probe field transverse width. We use the continuous atomic operators $\hat{\sigma}_{\mu\nu}(z) \equiv \frac{1}{N_z} \sum_i^{N_z} |\mu\rangle_i \langle \nu|$ averaged over $N_z = \rho(z)\Delta z \gg 1$ atoms within a small interval Δz around position z [15]. These continuous operators obey the relations $\hat{\sigma}_{\mu\nu}(z) \hat{\sigma}_{\nu'\mu'}(z') = \hat{\sigma}_{\mu\mu'}(z) \delta_{\nu\nu'} \delta(z - z')/\rho(z)$. We work in the frame rotating with the optical ω (probe field) and ω_c (auxiliary coupling field) frequencies, and the microwave ω_{ud} (spin-transition) frequency, as detailed in the next paragraph. Then the energy of the excited atomic level $|e\rangle$ is given by the detuning $\Delta = \omega - \omega_{eg}$ of the probe field from the transition resonance frequency ω_{eg} , and the energy of the Rydberg state $|r\rangle$ is defined via $\delta = (\omega_c + \omega_{ud}) - \omega_{re}$.

The last term of Eq. (1) describes the effective long-range interaction between the medium atoms and n_s spins at positions \mathbf{r}_j . These spins are represented by atoms with the Rydberg states $|u\rangle$ and $|d\rangle$ with the transition frequency ω_{ud} , and the spin lowering $\hat{\sigma}_- = |d\rangle\langle u|$ and rising $\hat{\sigma}_+ = |u\rangle\langle d|$ operators. The medium atoms and spins are coupled via the dipole-dipole interaction

$$D_{as} = \frac{1}{4\pi\epsilon_0\hbar} \left[\frac{\wp_{ri} \cdot \wp_{du}}{|\mathbf{R}|^3} - 3 \frac{(\wp_{ri} \cdot \mathbf{R})(\wp_{du} \cdot \mathbf{R})}{|\mathbf{R}|^5} \right],$$

where \wp_{ri} is the dipole moment of the atomic transition $|r\rangle \leftrightarrow |i\rangle$ between the Rydberg states $|r\rangle$ and $|i\rangle$, \wp_{du} is the dipole moment of the spin transition $|d\rangle \leftrightarrow |u\rangle$, and $\mathbf{R} \equiv (z\mathbf{e}_z - \mathbf{r})$ is the relative position vector between an atom at z and a spin at \mathbf{r} . To be specific, we consider a geometry of the system such that $\wp_{ri} \parallel \wp_{du} \perp \mathbf{R}$, i.e., \wp_{ri} and \wp_{du} are along the y (quantization) axis, and assume

that spin positions \mathbf{r}_j are away from the z axis, at $x_j > w$. Then the interaction $D_{as} = \frac{C_3}{|\mathbf{z}\mathbf{e}_z - \mathbf{r}_j|^3}$, with $C_3 \equiv \frac{\wp_{ri}\wp_{du}}{4\pi\epsilon_0\hbar}$, is finite for all z . The atomic excitation to the Rydberg state $|r\rangle$ is mediated by a non-resonant auxiliary coupling field of frequency ω_c which acts on the transition from the excited state $|e\rangle$ to the intermediate Rydberg state $|i\rangle$ with the Rabi frequency Ω_c and a large detuning $\Delta_c = \omega_c - \omega_{ie} = \omega_{ri} - \omega_{ud}$, $|\Delta_c| \gg |D_{as}|, \Omega_c$. Upon adiabatic elimination of the nonresonant state $|i\rangle$, we obtain the rate $D(\mathbf{z}\mathbf{e}_z - \mathbf{r}_j) = \frac{C_3\Omega_c/\Delta_c}{|\mathbf{z}\mathbf{e}_z - \mathbf{r}_j|^3}$ of the effective atom-spin exchange interaction. Due to negligible population of $|i\rangle$, we can then neglect the dipole-dipole interaction between the medium atoms, $D_{aa} \propto \frac{|\wp_{ri}\Omega_c|^2}{\Delta_c^2}$, which requires that $\left| \frac{\wp_{ri}}{\wp_{du}} \right| \ll \left| \frac{\Delta_c}{\Omega_c} \right|$.

Adiabatic elimination of $|i\rangle$ leads also to the ac Stark shift $\frac{\Omega_c^2}{\Delta_c}$ of level $|e\rangle$, which can be absorbed in the detuning Δ , and to the dipole-dipole interaction induced shift $\delta' = \frac{|D_{as}|^2}{-\Delta_c}$ of level $|r\rangle$, which should be added to δ . Looking ahead to the DEIT resonance in the vicinity of $\Delta \simeq -\delta$, we note that in order to be able to disregard the spatially varying shift δ' , we require it to be smaller than the DEIT linewidth $\frac{|D|^2}{|\gamma_e + i\delta|}$, where $\gamma_e \geq \frac{1}{2}\Gamma_e$ is the relaxation rate of the $\hat{\sigma}_{ge}$ coherence [15]. This leads to the condition $\frac{|\gamma_e + i\delta|}{\Omega_c} < \frac{\Omega_c}{|\Delta_c|} \ll 1$, i.e., the Rabi frequency Ω_c of the auxiliary field should be sufficiently larger than γ_e (setting $|\delta| < \gamma_e$ from now on), but still much smaller than $|\Delta_c|$.

We may assume that the n_s spin-atoms are placed in a small volume of size $\Delta r \ll x_s/3$ at position $\mathbf{r}_s = x_s\mathbf{e}_x + z_s\mathbf{e}_z$ with $x_s \gg w$ and $z_s \simeq L/2$, such that the interaction strength D_{as} does not change appreciably within Δr . All the spin-atoms then couple symmetrically to the medium atoms, forming thereby an effective large spin $J = \frac{1}{2}n_s$ of the Dicke model [44] with the symmetric states $|J, M_J\rangle$ corresponding to $J + M_J$ atoms in state $|u\rangle$ and the remaining $J - M_J$ atoms in $|d\rangle$, where $M_J = -J, \dots, J$ is the “magnetic” (spin projection) quantum number. With the collective spin-lowering $\hat{J}_- \equiv \sum_{j=1}^{n_s} \hat{\sigma}_-^{(j)}$ and rising $\hat{J}_+ \equiv \sum_{j=1}^{n_s} \hat{\sigma}_+^{(j)}$ operators, the last term of Hamiltonian (1) can then be written as $\int dz \rho(z) D(z) [\hat{\sigma}_{re}(z) \hat{J}_- + \hat{J}_+ \hat{\sigma}_{er}(z)]$, where $D(z) \equiv D(\mathbf{z}\mathbf{e}_z - \mathbf{r}_s)$. These operators obey standard spin-algebra relations: $\hat{J}_- |J, M_J\rangle = \sqrt{(J + M_J)(J - M_J + 1)} |J, M_J - 1\rangle$, $\hat{J}_+ |J, M_J\rangle = \sqrt{(J + M_J + 1)(J - M_J)} |J, M_J + 1\rangle$, $\hat{J}_z |J, M_J\rangle = M_J |J, M_J\rangle$, etc. Strictly speaking, we should also take into account the dipole-dipole exchange interactions between the spin-atoms, $\sum_{jj'} D_{ss}(\mathbf{r}_j - \mathbf{r}_{j'}) \hat{\sigma}_+^{(j)} \hat{\sigma}_-^{(j')}$, which generalizes the Dicke model to the Lipkin-Meshkov-Glick model [45]. In the special case of infinite range interaction, $D_{ss} = \text{const} \forall j, j'$, we obtain the Hamiltonian for the spin- J as $H_J = \hbar \hat{J}_z + D_{ss}(J^2 - \hat{J}_z^2)$, where \hbar is the effective magnetic field – detuning, in the present context. For simplicity, we neglect the dispersive (van der Waals)

interactions between the spin atoms. Below, our main concern is the case of at most a single spin-atom, $J = \frac{1}{2}$, but we will keep the notation generally applicable to any J , assuming for simplicity negligible interactions between spins, leading to equidistant (or degenerate, for $\hbar = 0$) spectrum of H_J . Such a situation can in principle be realized for a few spin atoms arranged in certain geometric configurations, e.g., on a line tilted by angle $\theta \simeq 54.7^\circ$ with respect to the direction of the dipole moment vector \wp_{du} (the y axis), since then $D_{ss} \propto |\wp_{du}|^2 (1 - \cos^2 \theta) \simeq 0$.

Note finally that if, in the last term of Hamiltonian (1), we replace the dipolar exchange operator $D(z) \hat{J}_-$ (and its Hermite conjugate) with a c-number Rabi frequency of a classical driving field Ω_d , this Hamiltonian will describe the usual EIT process [14, 15, 43].

B. Dynamics of the system

From Hamiltonian (1) we obtain the following Heisenberg equations for the relevant system operators:

$$\begin{aligned} (\partial_t + c\partial_z) \hat{\mathcal{E}}(z) &= i\eta N \hat{\sigma}_{ge}(z), \\ \partial_t \hat{\sigma}_{ge}(z) &= (i\Delta - \gamma_e) \hat{\sigma}_{ge}(z) \\ &\quad + i\eta \hat{\mathcal{E}}(z) [\hat{\sigma}_{gg}(z) - \hat{\sigma}_{ee}(z)] \\ &\quad - iD(z) \hat{\sigma}_{gr}(z) \hat{J}_+ + \hat{F}_{ge}, \\ \partial_t [\hat{\sigma}_{gr}(z) \hat{J}_+] &= [i(\Delta + \delta) - \gamma_r] \hat{\sigma}_{gr}(z) \hat{J}_+ \\ &\quad - i\eta \hat{\mathcal{E}}(z) \hat{\sigma}_{er}(z) \hat{J}_+ \\ &\quad - iD(z) \hat{\sigma}_{ge}(z) \hat{J}_+ \hat{J}_- + \hat{F}_{gr}, \end{aligned} \quad (2) \quad (3) \quad (4)$$

where $N = \rho L (\gg 1)$ is the total number of medium atoms (assuming uniform density), and γ_e and $\gamma_r (\ll \gamma_e)$ are the atomic coherence relaxation rates (with \hat{F}_{ge} and \hat{F}_{gr} the associated Langevin noise operators), while we ignore the decay of spins since Rydberg states $|u\rangle$ and $|d\rangle$ of spin-atoms are long-lived.

We consider adiabatic evolution of the system and drop the noise operators \hat{F} since they do not contribute to the dynamics of the atomic and normally-ordered field operators [14, 15]. With all the medium atoms prepared initially in the ground state $|g\rangle$ and a weak input probe field ($n_p \ll N$), we can neglect the depletion of $|g\rangle$ and set $\hat{\sigma}_{gg} \simeq \mathbb{1}$ and $\hat{\sigma}_{ee}, \hat{\sigma}_{er} \rightarrow 0$ in the above equations. From the stationary solution of Eqs. (3), (4) we then obtain the atomic coherence $\hat{\sigma}_{ge}$. Substituting it in the field propagation Eq. (2) without the time-derivative and comparing with $\partial_z \hat{\mathcal{E}} = i \frac{\omega}{2c} \chi \hat{\mathcal{E}}$ [43], we obtain the medium susceptibility

$$\hat{\chi}(z, \Delta) = \frac{2}{\omega} \frac{i\eta^2 N}{\gamma_e - i\Delta + \frac{|D(z)|^2 \hat{J}_+ \hat{J}_-}{\gamma_r - i(\Delta + \delta)}}. \quad (5)$$

Note that $2\eta^2 N/\omega = |\wp_{ge}|^2 \bar{\rho}/(\hbar\epsilon_0) = \frac{c}{\omega} \sigma_0 \bar{\rho} \Gamma_e$, where $\sigma_0 = 3\pi c^2/\omega^2$ is the atomic resonant absorption cross-section assuming the (population) decay rate $\Gamma_e = 2\gamma_e$ from state $|e\rangle$, and $\bar{\rho} = N/(w^2 L)$ is the volume density of

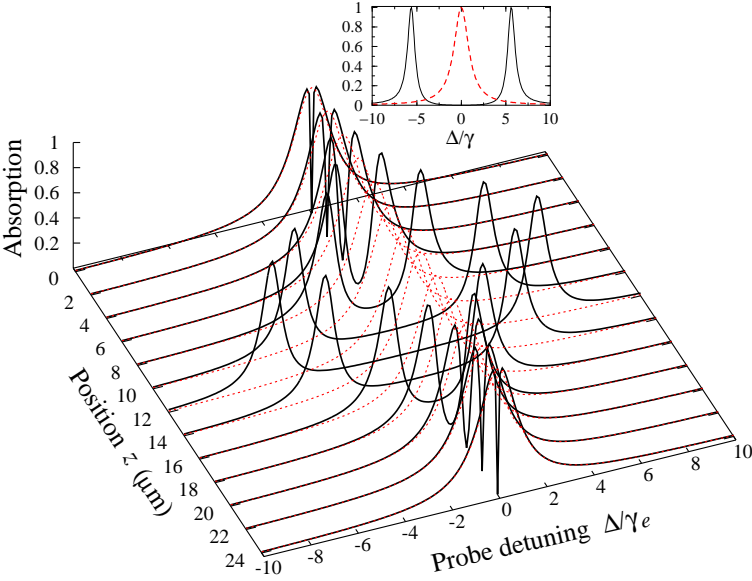


FIG. 2. Medium absorption $\frac{\omega}{2c} \text{Im}\langle\hat{\chi}(z, \Delta)\rangle$, in units of the resonant absorption coefficient $\sigma_0 \bar{\rho}$, as a function of position z and probe frequency (detuning) Δ , for $\langle\hat{J}_+ \hat{J}_-\rangle = 1$ (black solid lines) and $\langle\hat{J}_+ \hat{J}_-\rangle = 0$ (red dotted lines). Inset shows the DEIT and TLA (position independent) absorption spectra at $z = 12 \mu\text{m}$. We set $\gamma_r = 10^{-4} \gamma_e$ and $\delta = 0$, while $D(z)$ varies between $0.2\gamma_e$ (at $z = 0, L$) and $5.6\gamma_e$ (at $z = L/2$), as per parameters in Sec. III.

atoms. Equation (5) has the form of the usual EIT susceptibility [15, 43, 46], but with the square of the driving field Rabi frequency $|\Omega_d|^2$ replaced by that of the space-dependent dipolar exchange operator, $|D(z)|^2 \hat{J}_+ \hat{J}_-$. In Fig. 2 we show the imaginary part of medium susceptibility, Eq. (5), responsible for the probe absorption, as a function of probe frequency, at different spatial positions of the medium. In the absence of spin atom(s), $\langle\hat{J}_+ \hat{J}_-\rangle = 0$, the absorption spectrum is the usual Lorentzian of the TLA medium, while in the presence of a spin atom prepared in the spin-up state $|u\rangle$, such that $\langle\hat{J}_+ \hat{J}_-\rangle = 1$, the medium exhibits position-dependent EIT-like spectrum for a single probe photon (see below).

Using the expansion of $\hat{\chi}$ to first order in probe frequency around Δ [43], we can now write the propagation equation for the probe pulse amplitude as

$$(\partial_t + \hat{v}_g \partial_z) \hat{\mathcal{E}}(z, t) = i \frac{\omega}{2} \hat{\chi} \hat{\mathcal{E}}(z, t), \quad (6)$$

where $\hat{v}_g(z) = c[1 + \frac{\omega}{2} \frac{\partial}{\partial \Delta} \text{Re}\hat{\chi}(z, \Delta)]^{-1}$ is the group velocity [15, 43], which, as the susceptibility, is z -dependent and operator-valued quantity. We are concerned with the dynamics of probe field with the carrier frequency near the DEIT (EIT) resonance $\Delta = -\delta$, assuming the EIT-like condition $|\gamma_e + i\delta|\gamma_r \ll |D(z)|^2 \forall z \in [0, L]$. The group velocity is then

$$\hat{v}_g(z) = \frac{c}{1 + \frac{\eta^2 N}{|D(z)|^2 \hat{J}_+ \hat{J}_-}} \simeq c \frac{|D(z)|^2 \hat{J}_+ \hat{J}_-}{\eta^2 N} \ll c, \quad (7)$$

provided $\langle\hat{J}_+ \hat{J}_-\rangle \neq 0$ (see below), and assuming that the collective atom-field coupling $\eta^2 N$ is larger than the single atom-spin coupling $|D(z)|^2$ which remains finite even at $z \simeq L/2$ due to the spin position $x_s > w$. The propagation Eq. (6), supplemented with Eqs. (5) and (7), is the central result of this paper. Before we discuss its implications, however, we should establish the connection between the value of spin operator $\hat{J}_+ \hat{J}_-$ (for the given initial spin J) and the number of probe photons inside the medium $\hat{n}_p(t) = \frac{1}{L} \int_0^L dz \hat{\mathcal{E}}^\dagger(z, t) \hat{\mathcal{E}}(z, t)$.

Using Eq. (2) and

$$\partial_t \hat{\sigma}_{gg}(z) = i\eta \hat{\mathcal{E}}^\dagger(z) \hat{\sigma}_{ge}(z) + \text{H.c.}, \quad (8)$$

$$\begin{aligned} \partial_t \hat{J}_z &= \int dz \rho [iD(z) \hat{\sigma}_{re}(z) \hat{J}_- + \text{H.c.}] \\ &= - \int dz \rho \partial_t \hat{\sigma}_{rr}(z), \end{aligned} \quad (9)$$

and taking into account that $\partial_t [\hat{\sigma}_{gg} + \hat{\sigma}_{rr}] = 0$, since under the DEIT (EIT) resonance and adiabatic evolution the excited state $|e\rangle$ is never populated, $\hat{\sigma}_{ee} = 0$ [15, 43], we obtain that $\partial_t [\hat{n}_p - \hat{J}_z] = \frac{c}{L} [\hat{\mathcal{E}}^\dagger(0) \hat{\mathcal{E}}(0) - \hat{\mathcal{E}}^\dagger(L) \hat{\mathcal{E}}(L)]$ is determined by the difference of the flux of probe photons entering and leaving the medium at $z = 0$ and $z = L$, respectively. Next, from $\eta \hat{\mathcal{E}}(z) = D(z) \hat{\sigma}_{gr}(z) \hat{J}_+$ [cf. Eq. (3) with $\hat{\sigma}_{ge}, \hat{\sigma}_{ee} = 0$] we have $\eta^2 \hat{n}_p = \frac{1}{L} \int_0^L dz |D(z)|^2 \hat{\sigma}_{rr}(z) \hat{J}_- \hat{J}_+$. We assume that the initial spin J ($n_s = 2J$ spin atoms) is prepared in state $|J, J\rangle$ (all spin-atoms in state $|u\rangle$). Using the equality $\hat{J}_- \hat{J}_+ = (J - \hat{J}_z)(J + \hat{J}_z + 1)$, after a little algebra we obtain the approximate expression

$$\eta^2 N \hat{n}_p \approx 2J \bar{D}^2 (J - \hat{J}_z), \quad (10)$$

where we used $\int_0^L dz \rho \hat{\sigma}_{rr}(z) = (J - \hat{J}_z)$ assuming that $\hat{\sigma}_{rr}(z)$ is a slowly varying function of z in comparison to $|D(z)|^2$ with the mean value $\bar{D}^2 = \frac{1}{L} \int_0^L dz |D(z)|^2$. Recall that we positioned the spin J such that $\eta^2 N \gg |D(z)|^2 \forall z \in [0, L]$. Equation (10) therefore indicates that inside the medium nearly all of probe photons are converted into the spin (de-)excitations, $\hat{n}_p \ll (J - \hat{J}_z)$. We then obtain that

$$\begin{aligned} J - \hat{J}_z(t) &\simeq \frac{c}{L} \int_0^t [\hat{\mathcal{E}}^\dagger(0, t') \hat{\mathcal{E}}(0, t') - \hat{\mathcal{E}}^\dagger(L, t') \hat{\mathcal{E}}(L, t')] dt' \\ &\equiv \hat{n}_{p, \text{in}}(t) - \hat{n}_{p, \text{out}}(t). \end{aligned} \quad (11)$$

We can now deduce the response of the medium to the incoming probe photons. The $n_p = n_{p, \text{in}} - n_{p, \text{out}}$ photons, that already entered the medium but not yet left it, are coherently converted into the atomic Rydberg excitations $|r\rangle$ with simultaneous flip of n_p spin-atoms from state $|u\rangle$ to state $|d\rangle$, corresponding to the spin state $|J, J - n_p\rangle$. Operator $\hat{J}_+ \hat{J}_-$ acting on that state leads to $(n_s - n_p)(n_p + 1)$ which is non-zero if $n_s > n_p$. Then the next probe photon entering the medium sees vanishing susceptibility, since in Eq. (5) the last term in denominator diverges under the DEIT (EIT) conditions. That

$(n_p + 1)$ th probe photon propagates in the DEIT medium without absorption and with the n_p -dependent group velocity $v_g^{(n_p+1)}(z) = c|D(z)|^2(n_s - n_p)(n_p + 1)/(\eta^2 N)$ as per Eq. (7). We note parenthetically that if $n_s \gg n_p$ the large spin- J behaves as a harmonic oscillator and the group velocity depends nearly linearly on n_p – a situation similar to VIT with Λ -atoms in a cavity [18]. On the other hand, if $n_s \leq n_p$, the susceptibility of Eq. (5) reduces to that of the resonant TLA medium ($|\Delta| < \gamma_e$). Equation (6) then leads to linear absorption of the incoming probe photon, $\hat{\mathcal{E}}^\dagger(z)\hat{\mathcal{E}}(z) = \hat{\mathcal{E}}^\dagger(0)\hat{\mathcal{E}}(0)e^{-\kappa z}$, with the (intensity) absorption coefficient $\kappa = 2\sigma_0\bar{\rho}$ [43]. Thus the DEIT medium behaves as a photon number filter, transmitting up to $n_p \leq n_s$ probe photons at a time, given the number n_s of spin-atoms prepared in state $|u\rangle$.

Perhaps the most experimentally relevant and practically interesting situation of a single-photon filter or a transistor is realized for a single spin playing the role of a gate: For $n_s = 0$ the medium is strongly absorbing for the incoming probe photons, with the optical depth $\text{OD} = \kappa L$ which can be large enough in the medium of sufficient length L (see below); For $n_s = 1$ a single spin-atom in state $|u\rangle$ makes the medium transparent for one, and no more than one, probe photon at a time.

III. EXPERIMENTAL CONSIDERATIONS AND CONCLUSIONS

The system discussed above can be realized experimentally with currently available setups for Rydberg EIT with alkali atoms [25–27, 31, 36–38]. As a specific example, we may consider an ensemble of cold Rb atoms in an elongated trap of length $L \simeq 25 \mu\text{m}$. The ground $|g\rangle$ and excited $|e\rangle$ states of the medium atoms would correspond to suitable sublevels of the $5S_{1/2}$ and $5P_{1/2}$ (or $5P_{3/2}$) electronic states, with $\Gamma_e \simeq 2\pi \times 6 \text{ MHz}$, while the Rydberg states are $|i\rangle = |nS_{1/2}, M_J = 1/2\rangle$ and $|r\rangle = |nP_{3/2}, M_J = 1/2\rangle$ with the principal quantum number $n \simeq 82$, and the quantization direction is taken along the y axis ($|i\rangle \leftrightarrow |r\rangle$ is a π -transition, $\Delta M_J = 0$). The spin atom(s) can then be prepared by focused laser beam(s) in state $|u\rangle = |(n' + 1)S_{1/2}, M_J = 1/2\rangle$ with strong dipole transition to state $|d\rangle = |n'P_{1/2}, M_J = 1/2\rangle$ ($\Delta M_J = 0$). A static external electric or magnetic field can lift the degeneracy of the Rydberg M_J states of the spin and medium atoms to ensure that only $\Delta M_J = 0$ transitions are resonantly coupled via the dipole-dipole exchange interaction, in the presence of the auxiliary coupling field with the appropriate frequency ω_c to satisfy the two-photon resonance condition $|\delta| \ll \gamma_e$ ($\delta \simeq -\Delta$). With the quantum defects $\delta_S = 3.131$ and $\delta_P = 2.4565$ for the Rb S and P states [1], we chose $n' = 86$ such that the transition $|i\rangle \rightarrow |r\rangle$ is appropriately detuned (by $\Delta_c \simeq 10\Omega_c \simeq 2\pi \times 0.44 \text{ GHz}$) from the $|u\rangle \rightarrow |d\rangle$ transition resonance. Calculation of the transition dipole moments \wp_{ri} and \wp_{du} , involving the radial [47] and angular parts, leads to the coefficient $C_3 = 2\pi \times 10.8 \text{ GHz } \mu\text{m}^3$.

Then the DEIT linewidth $\delta\omega_{\text{DEIT}} = |D(R)|^2/\gamma_e$ of the medium atoms can be large enough, $\delta\omega_{\text{DEIT}} \gtrsim 2\pi \times 10^5 \text{ Hz}$ at a distance $R \lesssim d_t = 12.5 \mu\text{m}$ from the spin atoms, which permits the medium lengths $L \simeq 2d_t$. We take moderate atomic density $\bar{\rho} = 10^{12} \text{ cm}^{-3}$ at which the mean interatomic separations is larger than the size of the $n \sim 80$ Rydberg electron orbit, so as to avoid excessive decoherence due to electron collisions with the ground state atoms [41]. With $L \simeq 25 \mu\text{m}$ we then obtain optical depth $\text{OD} \simeq 7$, which would yield complete absorption of the probe photon(s) in the absence of DEIT, or saturation thereof by the previous $n_p = n_s$ photons. Decay and dephasing of Rydberg states of the medium and spin atoms, with the typical rate γ_r of several tens of kHz [25–29], will degrade DEIT and lead to a small probability of absorption of the probe photon(s), $\int_0^L dz \frac{z}{c} \text{Im}\langle\hat{\chi}(z)\rangle \simeq 10^{-2}$, during propagation through the medium. Several alternative choices of suitable atomic states and species are also possible. This will permit implementation of an efficient photon number switch as described above.

Note finally that in the above analysis we have neglected van der Waals interactions between the $|r\rangle$ state atoms. This simplification can be justified for a few probe photons simultaneously present in the medium, such that the mean distance between the photons is smaller than the van der Waals blockade distance $d_b = \sqrt[6]{C_6/\delta\omega_{\text{DEIT}}}$ [24, 28–30]. Due to the R dependence of the DEIT linewidth, $d_b = \sqrt[6]{\frac{C_6\gamma_e\Delta^2}{C_3^2\Omega_c^2}}|R|$ varies between $\sim 3 \mu\text{m}$ in the center ($z \sim L/2$) and $\sim 10 \mu\text{m}$ at the edges ($z \sim 0, L$) of the medium. Hence, even for many spin atoms $n_s > 1$, the number of probe photons in the medium is realistically limited to $n_p \leq 3$. Obviously, van der Waals interaction does not affect the performance of the single-photon switch, $n_s = 1$ or 0.

It would be interesting to consider an extended system with evenly distributed spin atoms prepared in one of the spin states with overlapping dipolar exchange field affecting the medium atoms. Then the density of probe photons that can propagate in the medium without attenuation will not exceed the density of spin atoms. Even more intriguing would be to explore simultaneous interaction of the Rydberg spin-atoms arranged in a chain-like 1D configuration with the medium atoms, and among themselves via resonant excitation (or hole) hopping. This may lead in bound states of the spin-flips (magnons) and propagating probe photons subject to DEIT. Developing an appropriate theoretical many-body description is a challenge worth pursuing as such systems could serve as viable quantum simulators with quantum light fields and Rydberg atoms.

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